Dueling Policies: Systemic Risk Taxation versus Constructive Ambiguity∗

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Abstract

This paper investigates the interaction between two policy instruments for the banking sector, namely systemic risk taxation and constructive ambiguity about bailout policy. It is generally acknowledged that bailout expectations may induce moral hazard in the form of excessive risk taking. This issue implies that it can be optimal for the financial regulator to appear ambiguous about bailout policies. Systemic risk taxation is an instrument intended to directly induce banks to prefer uncorrelated investments, leading to lower systemic risk formation. However, I find that systemic risk taxation may also inform banks about the regulator’s objective to ensure financial stability and thereby its bailout policy. Results indicate a trade-off between systemic risk taxation and constructive ambiguity. This trade-off highlights the importance to consider policies’ interdependence when evaluating their effectiveness in a context where the regulator’s objective is to maintain financial stability.

JEL codes: D83, G01, G18
Keywords: Systemic risk taxation, constructive ambiguity, bailout policy, financial crises

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1 Introduction

Two distinct objectives can only be achieved if at least two independent policy tools are at a regulator’s disposal according to Tinbergen. In a similar vein, the pursuit of one objective with multiple policy tools renders the tools dependent. In the aftermath of the Global Financial Crisis of 2007-2009 new reforms in prudential regulation have come to the fore to safeguard stability in the financial sector (Basel Committee of Banking Supervision, 2009a,b). Recent literature generally investigates the associated regulatory tools independently when their implications for financial stability are evaluated.¹ This study focuses on the interdependence between two such policy tools in a signaling game context: systemic risk taxation and constructive ambiguity in bailout support. Failing to account for the impact of a systemic risk tax on banks’ inferences about a regulator’s bailout decision leads to spurious conclusions about optimal systemic risk taxation. This result follows from banks forming and acting upon expectations about a financial regulator’s inclination to bail out banks during financial crises.

The approach in this paper has the flavor of the study by Farhi and Tirole (2012) in which the time-inconsistency problem of a financial regulator induces banks to coordinate with leverage choices on prospective bailout support. The main difference with their work lies in the focus of this paper on the signaling role of an introduced systemic risk tax and how the set tax contributes to interdependence among policy tools. With respect to endogenous policy decisions, previous studies have evaluated policy choices in light of speculative exchange rate attacks and anticipated interventions by the IMF (Drazen, 2000; Angeletos et al., 2006; Zwart, 2007).

The formation of systemic risk is driven in the model by banks’ endogenous investment choices, which determine the aggregate correlation structure among banks’ returns. The approach builds further on the model of Allen and Gale (2000) and Acharya (2009) and is close in line with the literature on inter-bank return dependence.² The main extension to Allen and Gale’s model is a regulator who sets a sys-

¹ Acharya et al. (2010) and Freixas and Rochet (2011) discuss the merit of a systemic risk tax levied on banks as compensation for explicit or implicit financial assistance by governments in times of financial crises. A second strategy regulators can adopt to deter banks’ risk-shifting behavior induced by prospective bailout support is to employ constructive ambiguity about bailout policies (Freixas, 1999; Kocherlakota and Shim, 2007; Shim, 2011).

²Maksimovic and Zechner (1991) show that the risk characteristics of firms’ cash flows are endogeneously determined by aggregate investment decisions in the industry. Financially distressed industry peers impose private costs on leverage of banks and may induce risk-shifting behavior (Shleifer and Vishny, 1992). Rajan (1994) connects managerial short-termism and reputation concerns to lowering of credit policy leniency when other banks are likely to do the same. Acharya and Yorulmazer (2007) consider return dependence driven by many banks coordinating on bailout prospects.
systemic risk tax for banks and faces a commitment problem in its decision to bail out distressed banks. When a substantial fraction of banks become financially distressed, the costs imposed on society of letting distressed banks fail can force the regulator to initiate bailout support.

In the presented model, banks have heterogeneous imperfect information about the conditions that force the regulator to initiate bailouts. The uncertainty about bailout prospects causes banks’ investment choices to become strategic complements (Acharya and Yorulmazer, 2007; Farhi and Tirole, 2012). As more banks become financially distressed the cost of their failure to society increases and this induces banks to believe that the regulator is likely to initiate bailout support. Preferences of banks may thereby shift towards having higher correlated returns across banks in order to trigger bailouts during banking crises. To analyze this coordination problem faced by banks I use the global games methodology (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2001, 2003). In this setting the implications of the systemic risk tax are twofold. First, an increase in the systemic risk tax may induce banks not to prefer joint correlated investments and thereby leads to lower systemic risk formation. Second, the tax has the effect of informing banks about the regulator’s optimal policy and thereby shapes banks’ expectations about future bailout decisions of the regulator.

Results suggest that banks’ knowledge of the tax level and their private information about the regulator’s inclination to initiate bailouts provide the means to infer the likelihood of receiving bailout support. In this light, a regulator that is perceived as highly inclined to initiate bailout support has an incentive to set a low tax in order to imitate a regulator that maintains a tough stance towards bailouts and does not need the tax to maintain financial stability, i.e. to curtail excessive risk taking by banks. Such a taxation strategy based on imitation generates the necessary uncertainty a weak regulator can exploit as constructive ambiguity. The uncertainty is driven by the observation of a low tax which does not allow banks to distinguish between these two regulator types and may thereby curtail coordination in risk taking by banks on bailout prospects. By taking the bailout policy into account I argue that in order to successfully curtail systemic risk formation with a systemic risk tax, the tax should be set independent of any future bailout policy in order to be effective.

With respect to policy implications, these results suggest that significant risks are associated with systemic risk taxation when the regulator is inclined to bail out banks during crises. In order to credibly install an effective taxation policy the costs incurred by the regulator to rescue a bank need to be high. These costs are not
necessarily limited to the actual absolute costs of a bank bailout, but also includes
the ease with which regulators and politicians can bail out banks. The effect of a
higher costs associated with a bailout have the effect that the regulator can generate
the necessary commitment to not bail out during a crisis if he desires to do so, because
in such events it is less likely that banks coordinate on bailout prospects.

The paper is organized as follows. Section 2 outlines the setup of the model. Section 3 presents the equilibrium results and section 4 provides a discussion and
interpretation of results. Section 5 contains concluding remarks. Derivations and
proofs are reserved for the appendix.

2 Model

The model builds upon the framework of Allen and Gale (2000) who explain finan-
cial bubbles and crises by risk-shifting behavior of a single investor in a one-period
setup. I follow Acharya (2009) but extend his model with a regulator and consider
many banks in order to study the formation of systemic risk based on inter-bank in-
teractions. A financial regulator can implement a systemic risk tax in the first stage of
the model and extend bailout support at the end of the first period. The presence of
the second stage allows banks to derive future value from a bailout. Figure 1 outlines
the stages of the game.

![Figure 1: Game Tree](image)

Banks and depositors are active in two periods that are bounded by three dates.
Depositors can consume the proceeds of their deposits at the end of the game. Banks
are owned and run by the same agents, they maximize their charter value over all
periods and readjust strategies at the start of each period. At the end of a period, the financial regulator evaluates whether it is optimal to bail out financially distressed banks.

2.1 Banks and Depositors

A continuum of banks of measure 1 exists, banks are indexed by $i$ and uniformly distributed over the unit interval, $[0,1]$. A continuum of depositors choose to supply funds for deposits at the start of each period denoted by $D_t$, where $t \in \{0,1,2\}$ indexes the dates. The amount $D_t$ is not fixed through the periods. When a bank fails the deposits are partially destroyed, and following favorable times positive returns are reinvested. The existence of a market maker is assumed who pools all funds available for deposits in a period and channels it to the banking sector. This assumption ensures that depositors’ wealth is diversified across all banks.

In a given period banks engage in perfect competition when they access the deposit market and the rate at which banks borrow funds from depositors is denoted by $r_t^{D}$. Depositors are paid at the end of a period. Furthermore, funds obtained from the deposit market constitute the only form of financing for banks. At the start of each of the two periods banks can decide on the allocation of raised deposits between risky assets and a safe asset as in the framework of Allen and Gale (2000). The appended Section A.1 provides a detailed exposition of the technologies underlying the safe and risky assets.

Payoffs – Banks set strategies at $t = 0$ and if they survive the first period also at $t = 1$. At these instances banks set the proportion of deposits to be invested in the safe asset $x_{i t}^{S}$ and the proportion to be invested in the risky assets $x_{i t}^{R}$. In addition, banks set the level of idiosyncratic volatility risk associated with the risky assets’ return $\sigma_{i t}$, and their choices of asset class $\rho_{i t}$. The latter choices determine the aggregate correlation structure of returns across banks.

A bank fails at the end of a period if the return on the risky asset $R_{i t+1}$ is insufficient. That is to say, if the return on the risky asset is below $r_{i t+1}^{C}$, the bank will be unable to repay depositors $r_{i t+1}^{D}D_t$. The critical return $r_{i t+1}^{C}$ can be inferred from the budget constraint:

$$r_{i t+1}^{C} = r_{i t+1}^{D} + (r_{i t+1}^{D} - r_{i t}^{S})x_{i t}^{S} / x_{i t}^{R}, \quad t \in \{0,1\}.$$

Rearranging the condition yields $r_{i t+1}^{C} = r_{i t+1}^{D} + (r_{i t+1}^{D} - r_{i t}^{S})x_{i t}^{S} / x_{i t}^{R}$. If $R_{i t+1} \geq r_{i t+1}^{C}$,
the pecuniary gain for the bank owners is denoted by

\[(R_{i+1}^{R} + r_{i+1}^{S} x_{i+1}^{S} - r_{i+1}^{D} (x_{i+1}^{R} + x_{i+1}^{S})) \times D_{i}.\]

Depositors would in this case receive a return of \(r_{i+1}^{D} (x_{i+1}^{R} + x_{i+1}^{S})\). In the event the return on risky assets is insufficient, \(R_{i+1} < r_{i+1}^{c}\), bank owners receive nothing and depositors receive the remaining asset value of the bank \((R_{i+1}^{R} + r_{i+1}^{S} x_{i+1}^{S}) \times D_{i}\).

Furthermore, I follow Allen and Gale (2000) and introduce costs associated with investing in risky assets. These costs can arise from monitoring efforts, administration and risk management and are non-pecuniary. Banks have a parsimonious cost function \(c : \mathbb{R}^{+} \rightarrow \mathbb{R}^{+}\) which features: \(c(0) = 0; c'(0) = 0; c'(x) > 0;\) and \(c''(x) > 0\).

Based on the previous discussions the expected payoff of bank \(i\) in a given period can be denoted by \(\varphi : [0, \sigma_{max}] \times [\mathbb{R}^{+}]^{4} \rightarrow \mathbb{R}\) and is defined by

\[
\varphi(\sigma_{i+1}, x_{i+1}^{R}, x_{i+1}^{S}, r_{i+1}^{D}, r_{i+1}^{S}) = \int_{r_{i+1}}^{r_{max}(\sigma_{i+1})} (R_{i+1}^{R} + r_{i+1}^{S} x_{i+1}^{S}
- r_{i+1}^{D} (x_{i+1}^{R} + x_{i+1}^{S}))db(\sigma_{i+1}, \cdot) - c(x_{i+1}^{R}),
\]

where \(b\) denotes the density of the risky assets’ return \(R_{i+1}\) which features mean reversion as outlined in the appended Section A.1.

### 2.2 Policy and systemic risk formation

Banks face two types of risk: idiosyncratic risk and systemic risk. The idiosyncratic risk component, volatility \(\sigma_{i+1}\), affects the probability of failure of the bank in a given period. The extent with which a bank’s performance is detrimentally affected by other bank failures is denoted as systemic risk. Bank managers do not observe other banks’ choices of asset classes, instead managers infer the likely aggregate action of banks through the expectation about whether to receive bailout support if financially distressed. Equilibrium choices are derived by means of backward induction.

**Objective of the regulator** – The regulator is concerned with maintaining financial stability. To this end the regulator can bail out financially distressed banks and levy a systemic risk tax. The bailout decision is determined on the basis of whether the cost of letting a bank fail outweigh the cost of saving a bank. The cost of letting bank \(i\) fail depends on bank \(i\)’s performance and the performance of other banks, \(-i\). These costs are denoted by \(C(i, -i)\). The costs \(C\) feature the notion that a single bank’s failure can potentially be absorbed by the banking sector, but multiple bank
failures may induce failure of other banks and thus increase the costs associated with their failure (Acharya et al., 2010). The cost the regulator perceives to be associated with rescuing a bank $\theta$ are constant and are not perfectly observed by banks.\(^3\)

The accuracy with which banks observe $\theta$ constitutes the source of the regulator’s ability to exercise constructive ambiguity about its bailout decision, which is made explicit in Section 3. The bailout decision is denoted by $q_i \in \{0, 1\}$, where $q_i = 1$ means a bailout for bank $i$ and $q_i = 0$ the converse. Hence, we can regard $\theta - C(i, -i)$ as the net benefit of not bailing out bank $i$.

The second tool of the regulator is to levy a systemic risk tax $\tau$. The aim of such a tax is to discourage banks from taking activities that have the potential to contribute to the formation of systemic risk. In this light recent literature suggests to tax banks on the basis of a metric that proxies the costs imposed on society by the bank’s failure (Acharya et al., 2010; Freixas and Rochet, 2011). To follow up on these studies, the tax is levied in this analysis on the probability of bank $i$ facing financial distress given that at least one other bank is distressed. The tax therefore aims to lower banks systemic exposure to one another. However, the tax also generates costs for the regulator. These costs are measured in terms of social welfare loss due to taxation. The tax implies an implicit transfer of funds from depositors to the regulator. These funds are thereby not invested in productive activities. These costs constitute a welfare cost to the regulator and are denoted by $\delta : [0, D_t] \to \mathbb{R}$ with the feature $\delta'(\tau) > 0$.

Based on the bailout decision, the benefits associated with the bailout policy and the costs of the tax the following objective function of the regulator can be formulated:

\[ (1 - q_i)(\theta - C(i, -i)) - \delta(\tau). \]

Furthermore, the bailout decision of the regulator follows endogenously from the notion of time inconsistency in bailout policy (O’Hara and Wayne, 1990; Brown and Dinç, 2005; Acharya and Yorulmazer, 2007). If the costs associated with letting a bank fail, $C$, exceed the costs of bailing out the bank, $\theta$, the regulator will always bail out the bank. Therefore the bailout decision can be interpreted as an indicator variable; $q_i \equiv 1\{\theta < C(i, -i)\}$, such that the objective function can be stated as:\(^4\)

\[ \max\{0, \theta - C(i, -i)\} - \delta(\tau). \]  

\(^3\)The costs associated with rescuing a bank are not necessarily limited to the actual absolute costs of a bank bailout, but also extend to the relative ease with which regulators and politicians can bail out financial institutions. The costs can therefore also reflect the extend with which funds have already been reserved for bailout funds.

\(^4\)Note that $q_i \equiv 1\{\theta < C(i, -i)\}$ equals 1 if $\theta < C(i, -i)$ and 0 otherwise.
Based on (2) banks form expectations about whether a bailout will be received when financially distressed. In this light banks conjecture the probability associated with the event $\theta < C(i, -i)$ to be the probability of receiving bailout support. Increases in their portfolio weight assigned to asset classes with high correlation $\rho_i$ results in a higher likelihood of joint failure. Consequently the probability of receiving bailout support increases as more banks opt for more weight of correlated asset classes in their portfolio. This constitutes the nature of the strategic complementarity in banks’ choice of asset class $\rho_i$.

**Implications for banks** – A bail out event enters each bank’s payoff schedule such that in the event of financial distress in the first period there exists a possibility to be allowed to operate in the second period. Likewise, other banks may receive bailout support when financially distressed and affects the payoff schedule of surviving banks.

In state $SF$ bank $i$ survives and at least one other bank is financially distressed. The financially distressed banks can be bailed out in this state, this occurs with probability $\hat{\pi}_{-i}$. The payoffs associated with either action of the regulator are denoted by the charter value $v^{SF}$ if a bailout is initiated, and $v^{SF}'$ if not. Based on lemma 5, see appendix Section A.3, we must have that $v^{SF} > v^{SF}'$, as a result of the negative spillover effect of banks’ failure on surviving banks. Turning to state $FF$, where the difference with $SF$ lies in bank $i$ being also financially distressed. In this event, the probability that $i$ will receive a bailout is now denoted by $\pi_{i}$, and that at least one other bank is saved is denoted by $\pi_{-i}$. Note that the positive charter values in this state are equal to those in $SF$. Since if bank $i$ would not be distressed the cost of any other bank failing would be strictly lower in comparison to the case where bank $i$ would be distressed. This conjecture will be made explicit when banks’ ex ante expectations about the costs of failure to society are defined in Section 3. A graphical depiction of the expected charter values for the various states and their corresponding probabilities of occurrence is presented in Figure 2. Furthermore, for ease of exposition if all but one bank are financially distressed, the regulator will not initiate a bailout policy.
By allowing the three bailout probabilities \((\pi_{i1}, \pi_{-i1} \text{ and } \hat{\pi}_{-i1})\) to be different, the regulator is assumed to implement bailouts for banks on a case by case basis, rather than a blanket bailout initiated for all banks. Nonetheless, the setup does not exclude the possibility of a uniform bailout across all distressed banks.

I use the notation \(A_1 := \{R_{i1} \geq r_{i1}^c\}\), and \(B_i := \bigcap_{j \in \{0,1\}/\{i\}} \{R_{j1} \geq r_{j1}^c\}\) for ease of exposition. These terms denote respectively the events in which bank \(i\) survives, and the survival of all other banks. Likewise the complements of these events can be denoted as \(A_{i1} := \{R_{i1} < r_{i1}^c\}\), failure of bank \(i\); and \(B_i^c := \bigcup_{j \in \{0,1\}/\{i\}} \{R_{j1} < r_{j1}^c\}\), the failure of at least one other bank. The optimal strategy profile for bank \(i\) for the first period can now be denoted as

\[
\begin{align*}
\sigma_i^* &= \text{arg max}_{\sigma_i, x_i^R, x_i^S, r^D_{it+1}, r^S_{it+1}} \{ v(\sigma_{it}, x_{it}^R, x_{it}^S, r^D_{it+1}, r^S_{it+1}) + \mathbb{P}^W(A_1 \cap B_i) v_{SS} \} \\
&\quad + \mathbb{P}^W(A_1 \cap B_i) (\hat{\pi}_{-i1} v_{SF} + (1 - \hat{\pi}_{-i1}) v'_{SF}) \\
&\quad + \mathbb{P}^W(A_1^c \cap B_i^c) \pi_{i1} (\pi_{-i1} v_{SF} + (1 - \pi_{-i1}) v'_{SF}) - \tau \},
\end{align*}
\]

where \(v(.)\) is defined as (11). Note that banks incorporate their expected charter value for the second period as well in determining their strategies. The operand in (3) is bank \(i\)'s charter value at the start of the first period and can be written conveniently.
as:

\[
V(\sigma_{it}, \rho_{it}, x_{it}^{R}, x_{it}^{S}, r_{it+1}^{D}, r_{it+1}^{S}) = v(.) + (1 - \mathbb{P}^{\mathcal{F}}(A_{i}^{c}) - \mathbb{P}^{\mathcal{F}}(B_{i}^{c}))\nu^{SS} + \mathbb{P}^{\mathcal{F}}(B_{i}^{c})(\hat{\pi}_{-i_{1}}\nu^{SF} + (1 - \pi_{-i_{1}})\nu^{SF}) \\
+ \mathbb{P}^{\mathcal{F}}(A_{i}^{c} \cap B_{i}^{c}) \left( \pi_{1}(\pi_{-i_{1}}\nu^{SF} + (1 - \pi_{-i_{1}})\nu^{SF}) \\
- (\hat{\pi}_{-i_{1}}\nu^{SF} + (1 - \hat{\pi}_{-i_{1}})\nu^{SF}) - \tau \right). 
\]

Note that the probability that bank \(i\) and at least one other bank fail, \(\mathbb{P}^{\mathcal{F}}(A_{i}^{c} \cap B_{i}^{c})\), is the only term in (4) that depends on the choice of asset class, \(\rho_{i}\). Bank \(i\) will set \(x_{it}^{R}, x_{it}^{S}\) in a similar vein as for the second period. The key difference lies in setting \(\sigma_{it}^{*}, \rho_{i}^{*}\), since these affect the distribution of returns for all banks and thereby \(\mathbb{P}^{\mathcal{F}}(.)\). However, \(\rho_{i}\), only affects the latter term such that the optimal choice \(\sigma_{it}^{*}\) can be expressed in terms of \(\rho_{i}^{*}\). It is assumed that the joint event where bank \(i\) fails and at least one other bank as well is increasing in banks' contribution to the overall formation of systemic risk, namely all chosen correlation terms.

The regulator charges the bank the amount \(\mathbb{P}^{\mathcal{F}}(A_{i}^{c} \cap B_{i}^{c})\tau\) as systemic risk tax at \(t = 0\) and after the banks set their choice of \(\rho_{i}\), such that the tax is incorporated in banks' decision on the asset class.

3 Equilibrium analysis

In order to evaluate the equilibrium decisions of banks at the start of the first period banks’ decisions with respect to their choice of asset class needs to be determined. Each bank solves in this respect:

\[
\rho^{*}(\pi_{i}) \in \arg \max_{\rho \in [\rho^{l}, \rho^{h}]} \{ \mathbb{P}^{\mathcal{F}}(A_{i}^{c} \cap B_{i}^{c}) (\pi_{i}b_{i} - c_{i} - \tau) \}, \quad \text{where} \\
b_{i} := \pi_{-i_{1}}\nu^{SF} + (1 - \pi_{-i_{1}})\nu^{SF} \\
c_{i} := \hat{\pi}_{-i_{1}}\nu^{SF} + (1 - \hat{\pi}_{-i_{1}})\nu^{SF}.
\]

Under the assumption of perfect information, each bank has knowledge with respect to the probability of receiving bailout support upon failure when at least one other bank is financially distressed. The optimal strategy for choice of asset class can then be expressed as:

\[
\rho^{*}(\pi_{i}) = \rho^{h}1\{\pi_{i} > (c_{i} + \tau)/b_{i}\} + \rho^{l}(1 - 1\{\pi_{i} > (c_{i} + \tau)/b_{i}\}). \quad (5)
\]
In case the bailout probability $\pi_i$ depends on the aggregate correlation structure, banks set $\rho^* = \rho^b$. This result is driven by the complementary nature of the choice of asset class. As more banks opt to assign more weight to the asset class prone to be correlated, the likelihood of joint failure of banks increases at the end of the first period. This reinforces banks’ incentives to opt for more correlated asset classes, because as more banks fail jointly the regulator is more inclined to initiate bailout support. Hence, the strategic complementarity in choice of asset class results in multiplicity of equilibria, and implies either all banks choose $\rho^b$ or low $\rho^l$ (Morris and Shin, 2003).

To avoid multiplicity of equilibria as a result of strategic complementarity I consider a noisy perturbation in banks’ knowledge about the cost for the regulator to initiate bailout support $\theta$. The approach follows the global games methodology and relates closely to the works of Morris and Shin (2003), and Angeletos et al. (2006). The regulator is given the option to reveal the actual value of the bailout costs, $\theta$, but this course of action is not optimal for the regulator. The imperfect knowledge of banks concerning $\theta$ generates the constructive ambiguity technology the regulator can exploit. The perturbation maintains strategic complementarily in banks’ asset class choices but introduces additional uncertainty for banks concerning regulatory bailout policy. To explicitly incorporate constructive ambiguity endogenously, rather than assuming constructive ambiguity to be present has the advantage that signalling effects of other policy tools on this strategy can be investigated.\footnote{Assuming constructive ambiguity to hold at all times conditions the analysis on this presumption, such that interactions between policy tools and constructive ambiguity cannot be evaluated.}

To evaluate the signaling role of a systemic risk tax, Bayesian-Nash equilibria are considered for two cases. First, a benchmark pooling equilibrium is considered where the systemic risk tax is fixed and cannot be changed by the regulator. This approach bears similarity to the manner in which Morris and Shin (2003) solve their model on currency crises. In the second case, the regulator is allowed to change the systemic risk tax which results in a signaling game. To this end, let $\tau(\theta)$ be the systemic risk tax set by the regulator with bailout costs $\theta \in \Theta \subset \mathbb{R}$. Furthermore, let $\rho^*(\xi_i, \tau(\theta))$ be bank $i$’s equilibrium choice of asset class based on received signal $\xi_i \in \Xi$ about $\theta$ and observed tax $\tau$; let $\rho(\theta, \tau)$ be an array which contains all banks’ asset class choices. Last, let $\mu(\theta < \theta' | \xi_i, \tau)$ denote bank $i$’s posterior distribution of $\theta$, i.e. bank $i$’s beliefs about the regulator’s type. With the posterior distribution banks infer the probability of receiving bailout support if financially distressed, $\pi_i \equiv \mu(\theta < \mathbb{E}^H(C(i, -i))|\xi_i, \tau)$ and $\mu(\mathbb{E}^H(C(i, -i))|\xi_i, \tau)$ for short. These expressions
allow for an equilibrium definition that characterizes the main actions by banks and the regulator in the first period of the model.

**Definition 1 (Equilibrium characterization)** – An equilibrium consists of a strategy for the regulator of type $\theta \in \Theta \subset \mathbb{R}$, $\tau : \Theta \to [0, D_t]$, a cumulative distribution function $\mu : \mathbb{R}^2 \times [0, D_t] \to [0, 1]$, and a strategy for banks $\rho : \mathbb{R} \times [0, D_t] \to [\rho^l, \rho^h]$, such that

$$\tau^*(\theta) \in \arg\max_{\tau \in [0, D_t]} W(\theta, \tau, \rho(\theta, \tau)),$$

$$\mu(\theta|\xi_i, \tau)$$ is obtained from Bayes’ rule, for any $\tau \in [0, D_t]$, and

$$\rho^*(\xi_i, \tau) \in \arg\max_{\rho \in [\rho^l, \rho^h]} \{\mathbb{P}[A_c \cap B_c](\mu(\mathbb{E}[C(i, -i)|\xi_i, \tau])b - c - \tau)\}.$$

Both for the case of a fixed tax and a strategic tax these equilibrium expressions are solved in the two subsequent sections.

With respect to banks’ choices for $\sigma^*_{i0}, x^{R*}_{i0}$ and $x^{S*}_{i0}$ at the start of the first period, these are in the same manner contingent on the banks choice of $\rho_{i0}$ as in Acharya (2009). Since these choices do not bear consequences for the main results of this paper they are omitted.\(^6\)

### 3.1 Fixed non-strategic systemic risk taxation

In this section the systemic risk tax is fixed, i.e. $\tau = \tau_0$. This implies $\tau^*(\theta) = \tau_0$ for all government types $\theta$. Banks remain therefore uninformed by the systemic risk tax about the cost of bailouts. Under imperfect knowledge, banks receive a noisy signal about the regulator’s cost of bailouts $\theta$, i.e. $\xi_i = \theta + \varepsilon_i$, where $\varepsilon \sim F$ with density $f$ and $\theta \sim G$ with density $g$. The signal and the prior knowledge of $G$ allow banks to infer a posterior of $\theta$. According to Bayes’ rule the posterior distribution of $\theta$ of bank $i$ can be expressed as:

$$\theta|\xi_i \sim \mu(\theta|\xi_i) = \frac{\int_{\tilde{\theta}}^{\theta} g(\tilde{\theta}) f(\xi_i - \tilde{\theta}) d\tilde{\theta}}{\int_{\theta_1}^{\theta_2} g(\theta) f(\xi_i - \theta) d\theta}.$$

Banks believe they will receive a bailout if the cost of letting them fail is sufficiently high, $\theta \leq \theta^*$. That is, the costs of bailing out the bank are lower than

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\(^6\)I refer the reader to proposition 2 in Acharya (2009) and the proof thereof for a detailed exposition of the choices of banks for these variables.
some critical cost of letting the bank fail. A bank with signal $\xi_i$ attaches probability $\mu(\theta^*|\xi_i)$ to the event of receiving a bailout when financially distressed, conditional on at least one other bank being financially distressed. Given this probability the optimal action of bank $i$ is denoted by $\rho^*(\mu(\theta^*|\xi_i))$. The average action across banks can be denoted under true $\theta$ by

$$\int_{\xi_i \in \Xi} \rho^*(\mu(\theta^*|\xi_i)) f(\xi_i - \theta) d\xi_i.$$  

The critical value $\theta^*$ corresponds with the ex ante expectation banks have of their cost of failure to the regulator $C(i, -i)$. Since, the outcome of $C(i, -i)$ is random, banks associate the critical level of bailout costs $\theta^*$ with $\mathbb{E}^H(C(i, -i))$, which is increasing in banks choice of $\rho_i$. As banks opt for asset classes prone to be correlated the joint failure probability increases and thus increases the expected cost to the regulator of letting a bank fail. For ease of exposition, I assume $\mathbb{E}^H(C(i, -i))$ to be linearly increasing in the average of banks’ asset class choices, such that the threshold condition $\theta^* = \mathbb{E}^H(C(i, -i))$ can be stated as:

$$\theta^* = \int_{\xi_i \in \Xi} \rho^*(\mu(\theta^*|\xi_i)) f(\xi_i - \theta^*) d\xi_i.$$  

(7)

Since the aggregate action of the banking system is decreasing in $\theta$, we have under the critical value $\theta^*$,

$$\int_{\xi_i \in \Xi} \rho^*(\mu(\theta^*|\xi_i)) f(\xi_i - \theta) d\xi_i > \theta.$$  

Since $f$ is continuous and monotonically decreasing in $\theta$, (7) is a sufficient condition for the existence of a unique threshold equilibrium.

Based on (7) and the monotonicity of $\mu$ there exists a unique signal $\xi^* := \xi(\theta^*, \bar{\pi})$ for which a bank attaches probability $\bar{\pi}$ to the event in which bailout support is received when distressed and at least one other bank is distressed. This signal is inferred from the condition

$$\mu(\theta^*|\xi(\theta^*, \bar{\pi})) = \bar{\pi}.$$  

Suppose bank $i$ receives such a signal $\xi(\theta^*, \bar{\pi})$ and believes with probability $\bar{\pi}$ to

---

7 The expected costs of failure follows the analogy with system wide shortfall (Acharya et al., 2010; Adrian and Brunnermeier, 2011).
receive bailout support when distressed and at least one other bank is distressed. In addition, suppose bank $j$ receives signal $\xi_j < \xi(\theta^*, \bar{\pi})$. It holds that $\mu(\theta^*|\xi_j) > \bar{\pi}$. Hence, the fraction of banks with stronger beliefs about receiving bailout support when at least one other bank fails relative to bank $i$ with belief $\bar{\pi}$ is denoted by:

$$\int_{\xi \in \Xi} f(\xi_i - \theta^*)d\xi_i = F(\xi_i(\theta^*, \bar{\pi}) - \theta^*)$$

Differentiation of this expression with respect to $\bar{\pi}$ results in the density of beliefs among banks, $\gamma(\bar{\pi}|\theta^*)$, under the true $\theta^*$. $\gamma$ is the density of beliefs about receiving bailout support distributed across the banking sector for a given type of regulator $\theta^*$. Combined with condition (7) and a change of variable $\xi_i$ to $\bar{\pi}$ allows to pin down the average action of the banks with respect to their choice of asset class, $\rho_i$:

$$\theta^* = \int_0^1 \rho^*(\pi)\gamma(\pi_i|\theta^*)d\pi_i.$$ 

Let $\hat{\theta} := \frac{\theta^* - \rho^l}{\rho^h - \rho^l}$ and we can state this more conveniently in terms of banks’ actions:

$$\frac{\theta^* - \rho^l}{\rho^h - \rho^l} = \frac{\int_0^1 \rho^*(\pi_i)\gamma(\pi_i|\theta^*)d\pi_i - \rho^l}{\rho^h - \rho^l}$$

$$\hat{\theta} = \int_0^1 \left\{ \pi > \frac{\epsilon + \tau_0}{b} \right\} \gamma(\pi_i|\theta^*)d\pi_i$$

Based on (5) the parameter $\hat{\theta}$ can be interpreted as an index and indicates the proportion of banks which choose $\rho^h$, whereas the remaining fraction chooses $\rho^l$.

At first it seems that (8) implies an increase in the systemic risk tax $\tau$ discourages banks from choosing $\rho^h$. The inference from this observation would be that the regulator is in a position to curtail systemic risk taxation by charging a sufficiently high systemic risk tax. However, a high systemic risk tax also poses a cost to the regulator since it reduces the investment opportunity set of depositors and bank owners. This potential reduction in wealth due to a high tax enters the regulator’s objective function (2) through the cost $\delta$. The result of this inference would be that the regulator can set the systemic risk tax in manner deemed optimal.

According to (8) the regulator has no incentive to reveal $\theta$ to the banks in order to keep an ambiguous bailout policy. Since this would result in the perfect information
case where all banks can coordinate on the prospect of receiving bailout support. In
the subsequent section the systemic risk tax can be chosen freely by the regulator and
in addition to being a cost to banks the tax acts as a signal that reflects the regulator’s
optimal policy choice with respect to bailout policy.

3.2 Strategic systemic risk taxation

The main result of this section is the derivation of a limited set of regulator types
for which it is optimal to levy a systemic risk tax to induce banks not to coordinate
on bailout prospects, i.e. not to choose $\rho^h$. If the costs associated with an increased
dead-weight loss due to a high tax $\tau > \tau$ exceed the net costs of letting a bank fail the
regulator finds the low tax, $\tau$, the optimal choice of systemic risk taxation. Likewise,
the costs of letting banks fail can be too low to justify a higher tax due to the ensuing
welfare loss. This contrasts the comparative static derived from (8), which seems to
suggest that a high systemic risk tax results in lower systemic risk formation for any
type of regulator.

Two equilibria are identified in the setting where the regulator can choose $\tau \in
[0, D_t]$. One in which banks are unresponsive to a higher systemic risk tax, a so-
called pooling equilibrium; and one in which a subset of regulator types benefits
from setting a higher systemic risk tax.

Proposition 2 (Equilibria with strategic systemic risk taxation) – When the regu-
lator is able to set $\tau \in [\tau, D_t]$ (with $\tau > 0$) two equilibria can be identified for the first
period of the model, a pooling equilibrium (I.) and a semi-separating equilibrium (II.):

I. There exists an equilibrium in which the regulator sets $\tau(\theta) = \tau$, $\forall \theta \in \Theta$. Banks’
optimal choice, and the regulator’s bailout decision are respectively given by:

\[ \rho^*(\xi, \tau) = \begin{cases} 
\rho^b & \text{if } \xi < \xi^*, \\
\rho^l & \text{otherwise}; 
\end{cases} \]

\[ q_i(\theta) = \begin{cases} 
1 & \text{if } \theta < \theta^* \\
0 & \text{otherwise}. 
\end{cases} \]

Where the private signal $\xi^* = \xi(\theta^*, \cdot)$, the critical cost of initiating no bailout $\theta^*$
and index $\hat{\theta}$, in (8), are the same as in section 3.1.

II. When the regulator sets $\tau(\theta) \in [\tau, \hat{\tau}]$, with $\hat{\tau} < D_t$ there exists an equilibrium in
which

\[
\tau(\theta) = \begin{cases} 
\tau^* & \text{if } \theta \in [\hat{\theta}, \hat{\theta}] \subset \Theta \\
\hat{\tau} & \text{if } \theta \notin [\hat{\theta}, \hat{\theta}]; 
\end{cases}
\]

\[
\rho^*(\xi_i, \tau) = \begin{cases} 
\rho^h & \text{if } \xi_i < -\infty \text{ or } (\xi_i, \tau) < (\xi(\hat{\theta}), \tau^*) \\
\rho^l & \text{otherwise}; 
\end{cases}
\]

\[
q_i(\theta) = \begin{cases} 
1 & \text{if } \theta < \theta_0 \\
0 & \text{otherwise.} 
\end{cases}
\]

Where \( \underline{\theta} \leq \theta^* \leq \hat{\theta} \) and \( \underline{\tau} \leq \tau^* \leq \hat{\tau} \leq D_t \). Additionally, the equilibrium values \( \theta, \) and \( \hat{\theta} \) solve

\[
\theta = \delta(\tau^*) = \int_{\xi_i \in \Xi} \rho^*(\mu(\theta|\xi_i)) f(\xi_i - \theta) d\xi_i,
\]

where the last term denotes the average action across banks.

A derivation of the equilibria is appended.

The first equilibrium \( I \) denotes a pooling equilibrium in which banks’ are unresponsive to the regulator’s choice of \( \tau \). The optimal choice of taxation by the regulator in this case is \( \underline{\tau} \), since the tax will not have an effect on bank’s choices and setting it higher only results in a welfare loss. This equilibrium bears close similarity with the one derived in Section 3.1.

Regarding the second equilibrium, banks are responsive to the regulator’s tax. It is in the interest of the regulator not to raise the tax beyond \( \tau^* \). A higher tax only results in additional costs to the regulator while banks would have already been successfully deterred in their choice for the correlated asset class \( \rho^h \) for a tax \( \tau^* \). The regulator’s choice \( \tau^* \) is dominated by the lower tax \( \underline{\tau} \) if the net costs to bail out banks is not sufficiently high; or if banks’ deem it likely for the regulator to initiate a bailout policy. This result is derived from the regulator’s objective function (2) and the average action of banks with respect to their choice of \( \rho \). Two conditions prevail that identify the types of regulator which do not prefer \( \tau \) over \( \underline{\tau} \).

\[
\int_{\xi_i \in \Xi} \rho^*(\mu(\theta|\xi_i)) f(\xi_i - \theta) d\xi_i < \delta(\tau^*) \quad (9a)
\]

\[
\theta < \delta(\tau^*) \quad (9b)
\]

Condition 9a yields a \( \hat{\theta} \) for which a regulator of type \( \theta > \hat{\theta} \) prefers to set \( \underline{\tau} \) since the costs of letting banks fail is not sufficiently high, and banks are likely to be deterred due to their private information about \( \theta > \hat{\theta} \), which induces a large fraction
of the banks to believe that a bailout is not a likely outcome during a crisis.

Condition (9b) tells us for $\theta < \theta = \delta(\tau^*)$ the regulator will have no incentive to set $\tau^*$ since banks will not be deterred in choosing $\rho^b$. In this case banks private information is likely to induce a large fraction of banks to believe that $\theta < \overline{\theta}$, such that a large fraction of banks believes to receive bailout support during a crisis event. However the fact that they observe $\tau$ for such a regulator renders banks unsure whether they face a regulator of a type $\theta < \overline{\theta}$ which is inclined to bail banks out, or a type $\theta > \hat{\theta}$ which is not inclined to do so. This ambiguity in type is the constructive ambiguity the regulator can create as a weak type by setting a tax $\tau$ to imitate the tougher regulator. Setting any other tax between $\tau$ and $\tau^*$ would immediately reveal that the regulator is of type $\theta < \overline{\theta}$ and would cause the banks to coordinate on bailout prospects.

4 Discussion of results

The main results of this paper are the regulator’s limitations to set optimally a systemic risk tax for banks when the tax reveals to banks the regulator’s inclination to initiate bailouts for banks. In section 3.1 the average action across banks in equilibrium (8) with respect to their choice of asset class, $\rho$, suggests that banks’ contribution to the formation of systemic risk can be mitigated by setting a higher tax regardless of the regulator’s bailout policy. This result contrasts the case where the tax is strategically set by the regulator as discussed in section 3.2.

Failing to account for the fact that the tax may reflect the regulator’s objective to safeguard financial stability may lead to spurious conclusions about optimal systemic risk taxation. This result is driven by the regulator’s objective to safeguard financial stability and renders the two policy tools interdependent. If the regulator’s inclination to initiate bailout support is high, $\theta < \overline{\theta}$, a higher tax may not prove to be sufficient to alter banks’ preferences for correlated assets, i.e. banks’ continue to coordinate and opt for correlated assets and set $\rho^b$. A higher tax results in a welfare loss. This loss is driven by the distortion in investment opportunities for banks’ owners and depositors.

The failure of intermediate levels of taxation, $\tau \in (\tau, \tau^*)$, to deter systemic risk formation is due to the private information banks have with respect to the regulator’s inclination to initiate bailout support. Combined with banks’ private information about the regulator the observation of an intermediate tax induces banks to believe that a sufficiently high tax is apparently sub optimal for the regulator they face, be-
cause the regulator is of a type that is likely to initiate bailout support. Therefore any tax of the level \( \tau \in (\tau, \tau^*) \) that does not reflect the action of a regulator that faces relatively high costs associated with the bailout of a bank reveals to banks the regulator’s type to be one that is highly inclined to bail out banks, i.e. \( \theta < \theta^* \).

This result implies that an intermediate tax is informative for banks and thereby cancels the effects of constructive ambiguity about bailout support to deter systemic risk formation. For regulators with a high inclination to initiate bailout support it is therefore optimal to not change the systemic risk tax in order to keep banks in the dark concerning the bailout policy. In this way the regulator with a high inclination to initiate bailout support imitates a regulator’s action which is not inclined to initiate bailout support. The ensuing ambiguity between types drives the source of uncertainty banks face that can be exploited by the regulator as constructive ambiguity in order to lower banks’ preferences for correlated assets.

Condition (9b) shows that the accuracy with which banks perceive the regulator’s inclination to initiate bailout support has no influence on the regulator’s lower threshold type \( \theta \) who is willing to set \( \tau^* \). From this we can infer that the above results are robust against changes in the specifications of the noisy perturbation. Hence, the set of regulators for which \( \theta < \theta^* \), the optimal taxation remains the original, or low tax \( \tau_\tau \), regardless of the distribution of private information across banks. The only requirement is that banks do not have perfect information about the regulator’s inclination to initiate bailout support, in order to avoid multiplicity. The upper limit of regulator types for whom it is optimal to set \( \tau^* \) is increasing in the average action of banks with respect to their choice of asset class. The result is that the equilibrium value of \( \theta \) is independent renders the results in this paper not prone to the critique made by Svensson (2006) about Morris and Shin’s (2002) argument that welfare can be increasing in the inaccuracy of private information.

5 Conclusion

The interdependence between systemic risk taxation and constructive ambiguity is relevant for their effectiveness because banks can adjust their risk profile after the implementation of a systemic risk tax scheme and before a bailout policy is executed. Banks’ risk-shifting behavior results in the model from the set tax and expected bailout policy that reflect the regulator’s objective to ensure financial stability. Since the level of taxation signals the regulator’s stance on how to maintain financial stability, banks learn through the perceived objective of the regulator the conditions
in which bailout support is likely to be initiated. I find that the introduction of a systemic risk tax therefore limits the degree of obfuscation the regulator can employ about its bailout policy for distressed banks. Conversely, if the regulator desires to maintain an ambiguous bailout policy, prospective risk shifting induced by the signaling effect of a systemic risk tax should be incorporated in the decision on the tax level. This finding suggests the existence of a trade-off between the two policy tools, and results from evaluating systemic risk taxation and ambiguous bailout programs in a joint framework.

The implications of this trade-off for macro-prudential policy inferences are two-fold. First, to evaluate the effectiveness of regulatory policy tools the policies need to be considered in a joint framework when they serve the single objective of maintaining stability of the financial industry. Failing to account for the interdependence between policy tools can lead to spurious outcomes with respect to policies’ effectiveness in handling financial crises. In the context of the considered framework I find for regulators with a high inclination to initiate bailout support that the introduction of a systemic risk tax can fail to successfully deter systemic risk formation. Second, the model adds the caveat that conditioning a framework and its outcomes on the assumption that one policy tool is effective at all times may give rise to spurious results as well. To maintain constructive ambiguity imposes restrictions on the regulator’s ability to set a systemic risk tax in the considered framework. If constructive ambiguity is assumed to hold at all times, the restrictions on the level of a systemic risk tax are ignored and can lead to undesired outcomes.

References


Appendix

A Consequences of bank failure

A.1 Safe and risky assets

Safe asset – The return on the safe asset is denoted by $r_S$ and materializes at the end of a period. Banks channel a proportion of their raised deposits to a sector in the economy that employs a neo-classical risk free technology. The firms in this sector and banks engage in perfect competition in accessing the market for the safe asset. This implies that $r_S$ equals the marginal rate of return on capital of the risk free technology. Let $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ denote the production technology of the risk free asset which features $f'(x) > 0; f''(x) < 0; \lim_{x \downarrow 0} f'(x) \rightarrow \infty$; and $\lim_{x \uparrow \infty} f'(x) = 0$. In this context $x$ denotes the total amount invested in the risk free asset. Perfect competition implies the equilibrium condition $r_S = f'(x)$.

Risky assets – The bank selects a risk profile based on idiosyncratic volatility risk $\sigma_i$ and preferred asset class $\rho_i$. Larger values for volatility risk $\sigma_i$ correspond with higher volatility in returns. For larger values of $\rho_i$ the bank opts for assets with higher correlation in returns. Based on the bank’s risk profile the risky assets yield a random return $R_{i,t+1} \sim b(\sigma_i, \rho_i)$, $t \in \{0, 1\}$, where the density $b(\sigma_i, \rho_i)$ belongs to a class of distributions $\mathcal{H}(\sigma_i, \rho_i, \sigma_{-it}, \rho_{-i})$ which feature mean-preserving spreads. Note that $\sigma_{-it}$ and $\rho_{-i}$ denote tuples that contain the composite actions of all other banks with respect to their choice of the risk parameters. As more banks opt for asset classes prone to be correlated, the overall performance of banks becomes more correlated. This conjecture reflects both empirical and theoretical findings in the literature on inter-bank return dependencies (Maksimovic and Zechner, 1991; Shleifer and Vishny, 1992; Rajan, 1994; Farhi and Tirole, 2012).

A.2 Size of banking sector

In order to construct a measure of the size of the banking sector at the start of the second period the set $\Omega$ is defined to contain all possible outcomes at $t = 1$ with respect to which banks survive and fail. This set can be interpreted as all possible outcomes of tossing a continuum of coins that flip with a fixed, possibly different, probability head or tail. Head and tail respectively correspond with the survival of a bank or failure. A size measure of the banking sector can then be constructed from the field $\mathcal{F}$ that contains the collection of all subsets of $\Omega$. The size of the banking sector in the second period can then be denoted by a function $s : \mathcal{F} \rightarrow [0, 1]$, where $s$ is assumed to be strictly increasing in the proportion of banks that survive the first period.
A.3 Equilibrium values at $t = 1$

The state $SF$ is characterized by the survival of bank $i$ and the failure of at least one bank in the first period. The optimal strategy profile of bank $i$ is denoted by:

$$
\sigma^*_i, x^R_i, x^S_i \in \arg\max_{\sigma_i, x^{R}, x^{S}} v(\sigma_i, x^R_i, x^S_i, r^D_i, r^S_i).
$$

(10)

For this state $v$ is defined by

$$
v_{SF}(.) = \int_{\tilde{r}_{i+1}}^{r_{\max}(\sigma_i)} (R_{it+1} x^R_i + r^S_i x^S_i - r^D_{it+1}(x^R_i + x^S_i)) db(\sigma_i, .) - c(x^R_i).
$$

(11)

For state $SS$ $v$ is expressed as $v^{SS}$, and can be regarded as a specific case of $v^{SF}$ in which all banks survive. A simplification is derived for $v$ before solving for the arguments in (10). This simplification is summarized in lemma 3, the derivation is appended.

Lemma 3 (return on deposits) – In any existing equilibrium $r^D_{i+1} = r^S_{i+1} = r_{i+1}$, i.e. the return on the safe asset is equal to the return demanded by depositors. This rate of return is equal to the critical return of banks. Hence, $r_{i+1} = r^D_{i+1} = r^S_{i+1} = r_{i+1}$.

Lemma 3 allows for the expression

$$
v_{SF}(.) = \int_{\tilde{r}_{i+1}}^{r_{\max}(\sigma_i)} (R_{it+1} x^R_i - r_{i+1}) x^R_i db(\sigma_i, .) - c(x^R_i).
$$

Bank $i$’s decision on the investment in the risky asset, $x^{R}_i(\sigma_i, r_{i+1})$, given $\sigma_i, r_{i+1}$, is implied by the first order condition:

$$
\int_{\tilde{r}_{i+1}}^{r_{\max}(\sigma_i)} (R_{it+1} - r_{i+1}) db(\sigma_i, .) = c'(x^{R}_i).
$$

Since density $b$ is from a family of mean-preserving spread densities the first order condition can be rearranged as

$$
\tilde{R}_{it+1} - c'(x^{R}_i) = r_{i+1} + \int_{0}^{r_{\max}(\sigma_i)} (R_{it+1} - r_{i+1}) db(\sigma_i, .),
$$

(12)

where $\tilde{R}_{it+1}$ denotes the mean return value. Note that the right hand side of (12) is non-negative. Hence, a solution for $x^{R}_i$ exists, provided that $\tilde{R}_{it+1} \geq c'(D_{i1})$.

Lemma 4 (Equilibrium existence at $t = 1$) – For state $SF$ the symmetric equilibrium $(x^{R}_i, \sigma^*_i, r^*_i)$ exists at $t = 1$, with $\sigma^*_i = \sigma^{max}$.

---

9Since $c''(x) > 0$, $\forall x$, $x^{R}_i$ denotes a global maximizer of $v$. 

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Lemma 4 summarizes the equilibrium for state SF at time \( t = 1 \). It can be shown for state SS that there exists a symmetric equilibrium as well. The proof is similar to that of Lemma 4, since SS is a special case with \( s = 1 \). That is to say, no banks have failed prior to arriving in state SS. Nonetheless the equilibrium values may be different for both states for different values of \( s \). Comparative statics associated with these two variables are of interest to infer the implications of bank failure for surviving banks’ charter values.

Lemma 5 (Comparative static for state “SF”) – Let the charter value of a bank in state SF be denoted by \( v^{SF} \). Furthermore, since the equilibrium at \( t = 1 \) is driven by \( r^{*}_{t+1} \), \( v^{SF} \) is solely expressed in terms of \( r^{*}_{t+1} \). For similar reasons, \( r^{*}_{t+1} \) is expressed in terms of \( s \). Then the following result holds:

\[
\frac{\partial v^{SF}(r_{t+1}(s))}{\partial s} > 0, \quad \forall s \in [0, 1].
\]

The comparative static in Lemma 5 reflects that if a group of banks fail in the first period a fraction of the deposit pool is destroyed. This raises the return on the safe asset and consequently banks substitute safe investments for risky investments. However, no profits are reaped in the marked for the safe asset due to perfect competition. As a result banks’ charter values decline. Conversely, if a smaller fraction of banks fail charter values increase.

### B Proofs

**Proof of Lemma 3 (Return on deposits):** \( r^{D}_{it+1} = r^{S}_{it+1} = r^{c}_{it+1} \).

Suppose \( r^{D}_{it+1} > r^{S}_{it+1} \geq 0 \). From (11) the critical return is given by

\[
r^{c}_{it+1} = r^{D}_{it+1} + (r^{D}_{it+1} - r^{S}_{it+1}) \frac{x^{S}_{it}}{x^{R}_{it}}.
\]

Based on (11) and the above expression, for \( r^{D}_{it+1} > r^{S}_{it+1} \geq 0 \) bank \( i \) has no demand for the safe asset. However, this implies that \( r^{S}_{it+1} = \lim_{x \to 0} f'(x) \to \infty \), a contradiction.

Suppose \( 0 \leq r^{D}_{it+1} < r^{S}_{it+1} \). This implies that \( r^{c}_{it+1} \) is increasing in \( x^{S}_{it} \) and (11) is increasing in \( x^{S}_{it} \). Therefore, bank \( i \) has an infinite demand for the safe asset. However, for a limited supply of deposits \( x^{R}_{it} + x^{S}_{it} \) the budget constraint or the short sale constraint would be violated, a contradiction. Hence, we are left to conclude that

\[
r^{D}_{it+1} = r^{S}_{it+1} = r^{c}_{it+1} \]

**Proof of Lemma 4 (Equilibrium existence):** For state SF the symmetric equilibrium \( (x^{R*}_{t}, \sigma^{*}_{t}, r^{*}_{t+1}) \) exists at \( t = 1 \), where \( \sigma^{*}_{t} = a^{max} \).
First, the choice for $x_{t}^{R*}$ is obtained by solving (12). This allows to define $x_{t}^{R*}$ to be a function of $\sigma_{it}, r_{t}$, such that $x_{t}^{R*} \equiv x_{t}^{R*}(\sigma_{it}, r_{t})$. Substitution in (11), and noting Lemma 3 along with first-order condition (12) yields:

$$\frac{\partial \psi(x_{it}^{R*}(\sigma_{it}, r_{t}), \sigma_{it}, r_{t})}{\partial \sigma_{it}} = x_{t}^{R*}(\sigma_{it}, r_{t}) \frac{\partial}{\partial \sigma_{it}} \int_{0}^{r_{t+1}} (r_{i+1} - R_{i+1}) db(\sigma_{it},.) > 0.$$  

The inequality on the right hand side holds because expected losses are increasing in $\sigma_{it}$ set by the bank. Independent of $r_{t+1}$ it is optimal for banks to set $\sigma_{t}^{*} = \sigma^{max}$.

The rate of return on the safe asset is now obtained by an application of Brouwer’s fixed point theorem (Mas-Colell, Whinston, and Green, 1995, p. 952). Note that the total amount invested in the risk free asset in state SF amounts to $sD_{t} - x_{t}^{R*}(\sigma^{max}, r_{t+1})$. Since $f_{t}$, the technology of the risk free asset, is $C^{2}$ and concave on the domain $\mathbb{R}^{+}$. Therefore, the unique equilibrium rate of return on the safe asset is obtained from the condition:

$$r_{t+1}^{*} = f'(sD_{t} - x_{t}^{R*}(\sigma^{max}, r_{t+1})).$$

Therefore, the equilibrium $(x_{t}^{R*}, \sigma_{t}^{*}, r_{t+1}^{*})$ exists at $t = 1$ for state SF. \[\square\]

**Proof of Lemma 5 (Comparative statics for state “SF”)**: For a bank in state SF at time $t = 1$ the following holds: $\frac{\partial \psi_{SF}(.)}{\partial s} > 0$.

For ease of exposition the subscript $\sigma^{max}$ is dropped for $x_{t}^{R*}$, such that $x_{t}^{R} := x_{t}^{R*}(r_{t+1})$. Since $x_{t}^{R*}$ is solely determined by (12) the derivative $\frac{dx_{t}^{R}(r_{t+1})}{dr_{t+1}}$ is obtained from total differentiating (12) and yields:

$$\frac{dx_{t}^{R}(r_{t+1})}{dr_{t+1}} = \frac{-1}{c''(x_{t}^{R}(r_{t+1}))} \left(1 - \frac{d}{dr_{t+1}} \int_{0}^{r_{t+1}} (r_{t+1} - R_{i+1}) db(\sigma_{it},.)\right)$$

$$= \frac{-1}{c''(x_{t}^{R}(r_{t+1}))} \left(1 - \mathbb{P}(R_{it+1} \leq r_{t+1})\right) < 0.$$  

The second step in the proof is to evaluate the effect of $r_{t+1}$ on the charter value. Combining the results of Lemma 3 and 4 in (11) yields:

$$\frac{d\psi(x_{it}^{R}(r_{t+1}), r_{t+1})}{dr_{it}} = -x_{t}^{R*}(r_{t+1}) \left(1 + \frac{d}{dr_{t+1}} \int_{0}^{r_{t+1}} (r_{t+1} - R_{i+1}) db(\sigma_{it},.)\right) < 0.$$  

In order to proof the first result about the effects of $s$ on the charter value of a bank in state SF let $r_{t+1}^{*} = r_{t+1}^{*}(s)$. Then from the equilibrium condition for the risk free rate we have

$$r_{t+1}^{*}(s) - f'(sD_{t} - x_{t}^{R*}(r_{t+1}(s))) = 0.$$  

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Applying the implicit function theorem then yields:

\[
\frac{dr_i^{*}(s)}{ds} = \frac{f''(sD_t - x_i^{R_t}(r_{i+1}))D_t}{1 + f''(sD_t - x_i^{R_t}(r_{i+1}))DR_{i+1}} < 0,
\]

where the denominator is strictly positive. Therefore,

\[
\frac{dv(x_i^{R_t}(r_{i+1}(s)), r_{i+1}(s))}{ds} = \frac{dv(x_i^{R_t}(r_{i+1}), r_{i+1})}{dr_{i+1}} \frac{dr_{i+1}(s)}{ds} > 0.
\]

\[\Box\]

**Proof of Proposition 2 (Strategic systemic risk tax):** The proofs for both equilibrium I and II are considered below.

**Equilibrium I.** - This equilibrium constitutes a pooling equilibrium. In this equilibrium all banks are unresponsive to the regulator's action with respect to \(\tau\). Therefore, a regulator of any type \(\theta\) finds it optimal to set \(\tau\), since the welfare costs associated with the tax \(\delta\) are increasing in \(\tau\). The monotonicity of \(\delta\) establishes the optimality of \(\tau\).

Since the tax \(\tau\) is set by all types of regulator, the tax is uninformative to banks about the regulator's type \(\theta\). Therefore, beliefs of banks about the costs of a bailout are pinned down in an equal manner as for (6), such that \(\mu(\theta|\xi_i, \tau) \equiv \mu(\theta|\xi_i)\). Hence, as in Section 3.1, banks believe under non-strategic taxation a bailout will be initiated if \(\theta < \hat{\theta}\). Additionally, as in the case with a fixed systemic risk tax the bank finds it optimal to choose \(\rho^b\) if \(\xi_i < \hat{\xi}\).

In case the regulator sets \(\tau > \bar{\tau}\) banks detect a deviation. Let \(\tilde{\Theta}(\tau)\) denote the set of regulators for which the deviation to \(\tau > \bar{\tau}\) is dominated in equilibrium. Then for any signal \(\xi_i\) we must have the following two conditions:

\[
\mu(\theta \in \tilde{\Theta}|\xi_i, \tau) = 1,
\]

\[
\mu(\theta \in \tilde{\Theta}|\xi_i, \tau) = 0, \text{ if } \Theta \notin \tilde{\Theta}(\tau).
\]

The two conditions form the natural restriction that beliefs should assign positive measure to types that could lead to signal \(\xi_i\), and require that beliefs assign zero measure to types for which \(\tau > \bar{\tau}\) is dominated in equilibrium.

The next step is to verify that the set of beliefs in equilibrium I. is non-empty for all \(\tau > \bar{\tau}\). Banks react to the signal by choosing \(\rho^l\) unless they are convinced the regulator must be of a low type, i.e. if \(\xi_i < -\infty\). Borrowing the conditions (9b) and (9a) from section 3.2 allows us to pin down the set of \(\theta\) which set \(\tau\) such that the strategy is dominated in equilibrium by \(\bar{\tau}\). \(\hat{\theta}\) solves:

\[
\hat{\theta} = \delta(\bar{\tau}) = \int_{\xi_i \in \Xi} \rho^*(\mu(\hat{\theta}|\xi_i))f(\xi_i - \hat{\theta})d\xi_i,
\]

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such that $\tilde{\Theta}(\tau) = \mathbb{R}$ for $\tau > \tilde{\tau}$, and $\tilde{\Theta}(\tilde{\tau}) = \mathbb{R} \setminus \{\hat{\theta}\}$. Therefore, the set of beliefs satisfying the above-mentioned conditions is non-empty for $\tau \geq \tilde{\tau}$. If $\tau \in (\tau, \tilde{\tau})$, then the set of regulator types for which this strategy is dominated is $\Theta \setminus [\hat{\theta}, \tilde{\theta}]$, where

$$\hat{\theta} = \delta(\tau) = \int_{\xi_i \in \Xi} \rho^*(\mu(\hat{\theta]|\xi_i)))f(\xi_i - \hat{\theta})d\xi_i.$$ 

Since $\underline{\theta} < \hat{\theta} < \bar{\theta}$, the set of types for which $\tau$ is dominated in equilibrium is $\tilde{\Theta}(\tau) = \mathbb{R} \setminus [\underline{\theta}, \bar{\theta}]$.

**Equilibrium II.** – In the second equilibrium, banks coordinate on the systemic risk tax. Banks take average action

$$\int_{\xi_i \in \Xi} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i,$$

when $\tau < \tau^* \in [\tau, \tilde{\tau}]$, and all banks choose $\rho^j$ as an optimal response when $\tau \geq \tau^*$. It is optimal for the regulator to choose $\tau$ if $\tau < \tau^*$, since for $\tau < \tau^*$ banks do not respond to the tax in which case it is optimal for the regulator to minimize welfare loss $\delta$. Banks are responsive to the tax when $\tau > \tau^*$, such that the regulator prefers to set $\tau^* > \tau$ in this case.

For $\theta < 0$ it is dominant for the regulator to set $\underline{\tau}$. However, when $\theta > 0$ the payoff of setting $\tau^*$ is $\theta - \delta(\tau)$, and for $\tau$ the payoff is

$$\max\left\{0, \theta - \int_{\xi_i \in \Xi} \rho^*(\mu(\theta|\xi_i))f(\xi_i - \theta)d\xi_i\right\}.$$ 

These payoffs illustrate that the costs of setting a higher $\tau^*$ should not exceed the costs to bail out banks, and if banks deem it sufficiently likely that no bailout policy will be initiated $\underline{\tau}$ is preferred over $\tau^*$. Based on the conditions, the regulator types which prefer $\tau^*$ over $\underline{\tau}$ have $\theta \in [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta}$ and $\bar{\theta}$ solve

$$\underline{\theta} = \delta(\tau^*) = \int_{\xi_i \in \Xi} \rho^*(\mu(\bar{\theta}|\xi_i))f(\xi_i - \bar{\theta})d\xi_i.$$ 

Banks’ beliefs are pinned down by Bayes’ rule but differ from equilibrium I, since whenever the regulator sets $\underline{\tau}$, the corresponding type $\theta \notin [\underline{\theta}, \bar{\theta}]$. Hence beliefs about a bailout policy being initiated conditional on observing $\underline{\tau}$ are

$$\mu(\theta|\xi_i, \underline{\tau}) \equiv \frac{\mu(\theta|\xi_i)}{1 - \mu(\theta|\xi_i) + \mu(\bar{\theta}|\xi_i)},$$

where $\mu(\theta|\xi_i)$ is defined as (6) and $\mu(\theta|\xi_i, \underline{\tau})$ is decreasing in $\xi_i$. The last monotonicity result ensures uniqueness of equilibrium for the case where banks respond to a strategically set systemic risk tax. Furthermore, the fact that $\Theta \equiv \mathbb{R}$ ensures that the
set of regulator types for which \( \tau \in (\tau, \tau^*) \) is dominated in equilibrium by \( \tau \) is a subset of \( \Theta \), since \( \Theta(\tau^*) = \mathbb{R} \setminus [\underline{\theta}, \bar{\theta}] \).

\[ \square \]